ANGEL Torispheric Head Design, using (2010 ASME PV Code Section VIII, div. 2, part 4 rules) CF flange window version NW100: full opening DRAFT D. Shuman, LBNL, Apr 17, 2011

These design rules require selecting trial dimensions and iterating to find acceptable stresses. Shown here are final dimensions from repeated iterations.

Torispheric head with multiple ports for sapphire windows in NW100 CF flanges. First step determine the following:

I.D.

$$D_i := 1.14m$$

Crown radius (largest size possible without using part 5, design by analysis)

constraints

$$L_{cr} \coloneqq 1.14m \qquad \qquad 0.7 \le \frac{L_{cr}}{D_i} \le 1.0 = 1 \quad \text{(true when conditional = 1)}$$

Knuckle radius, here chosen to maximize crown, smallest size that gives sufficient pressure capacity after increasing head thickness for reinforcement of nozzles. Use 0.17m otherwise

$$r_{kn} := .07m$$
 $\frac{r_{kn}}{D_i} \ge 0.06 = 1$

(c) Step 3 calculate:

thickness, this is an iterated value after going through part 4.5.10.1 (openings) further down in the document

$$t_{ts} \coloneqq 15 \text{mm} \qquad \qquad 20 \le \frac{L_{cr}}{t_{ts}} \le 2000 = 1 \qquad \qquad \text{initial thickness (no nozzle reinforcement) value: 4.9mm for } \\ \beta_{th} \coloneqq a\cos\left(\frac{0.5D_i - r_{kn}}{L_{cr} - r_{kn}}\right) \qquad \beta_{th} = 1.085 \, \text{rad} \\ \phi_{th} \coloneqq \frac{\sqrt{L_{cr} \cdot t_{ts}}}{r_{kn}} \qquad \phi_{th} = 1.868 \, \text{rad} \\ R_{th} \coloneqq \frac{0.5D_i - r_{kn}}{\cos(\beta_{th} - \phi_{th})} + r_{kn} \qquad \text{$<$$--disabled calculation, because conditional->} \quad \phi_{th} < \beta_{th} = 0 \quad \text{is false} \\ R_{th} \coloneqq 0.5D_i \qquad \text{$<$$--enabled calculation, because conditional->}} \quad \phi_{th} \ge \beta_{th} = 1 \quad \text{is true} \\ R_{th} = 0.57 \, \text{m} \qquad \qquad \text{$<$$--enabled calculation, because conditional->}}$$

(d) Step 4 compute:

$$C_{1ts} := 9.31 \left(\frac{r_{kn}}{D_i} \right) - 0.086 \qquad \qquad \text{for} \qquad \frac{r_{kn}}{D_i} \le 0.08 = 1$$

$$C_{1ts} := 0.692 \left(\frac{r_{kn}}{D_i} \right) + 0.605 \qquad C_{1ts} = 0.486 \qquad \qquad \text{for} \qquad \frac{r_{kn}}{D_i} \ge 0.08 = 0$$

$$C_{2ts} := 1.25 \qquad \qquad \text{for} \qquad \frac{r_{kn}}{D_i} \le 0.08 = 1$$

$$C_{2ts} := 1.46 - 2.6 \left(\frac{r_{kn}}{D_i}\right)^{\bullet} \qquad C_{2ts} = 1.25 \qquad \qquad \text{for} \qquad \frac{r_{kn}}{D_i} > 0.08 = 0$$

(e) Step 5, internal pressure expected to cause elastic buckling at knuckle

$$P_{eth} := \frac{C_{1ts} \cdot E \cdot t_{ts}^2}{C_{2ts} \cdot R_{th} \cdot (0.5R_{th} - r_{kn})}$$
 $P_{eth} = 728 \text{ bar}$

(f) Step 6, internal pressure expected to result in maximum stress (S_{ν}) at knuckle

$$\begin{split} C_{3ts} &\coloneqq S_{y_Ti_g3} & S_{y_Ti_g3} \coloneqq 55000 psi & \text{time independent} \\ P_y &\coloneqq \frac{C_{3ts} \cdot t_{ts}}{C_{2ts} \cdot R_{th} \cdot \left(0.5 \frac{R_{th}}{r_{kn}} - 1\right)} & P_y = 26 \, \text{bar} \\ \\ (g) \, \text{Step 7} & G_{th} &\coloneqq \frac{P_{eth}}{P_y} & G_{th} = 28.377 \\ P_{ck} &\coloneqq \left(\frac{0.77508 \cdot G_{th} - 0.20354 \cdot G_{th}^2 + 0.019274 \cdot G_{th}^3}{1 + 0.19014 G_{th} - 0.089534 G_{th}^2 + 0.0093965 G_{th}^3}\right) \cdot P_y & P_{ck} = 51 \, \text{bar} \end{split}$$

(h) Step 8

$$P_{ak} := \frac{P_{ck}}{1.5} \qquad P_{ak} = 34.25 \, \text{bar}$$

(i) Step 9

$$P_{ac} := \frac{2S_{max_Ti_g3_div2} \cdot 1}{\frac{L_{cr}}{t_{ts}} + 0.5}$$

$$P_{ac} = 49.1 \text{ bar}$$

(j) Step 10

$$P_a := min(P_{ak}, P_{ac})$$
 $P_a = 34.3 \, bar$ OK, min thickness is 4.9mm (no openings rkn=.17m)

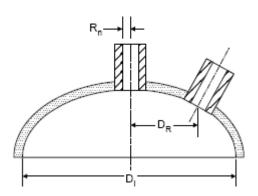
4.5.10.1 Radial Nozzle in formed head

We need to accommodate 76mm dia PMT's, having 64mm photocathode, but 7.75cm max dia., and we desire to fit PMT through opening in head, so let:

$$R_n := 3.95cm$$

We have no nozzle (flange does not count for reinforcement under div 1 and perhaps div 2

$$t_n := 0.01 mm$$
 (we need a non zero value for further calculations)



Considering nozzle as separate from head, to obtain a trial thickness

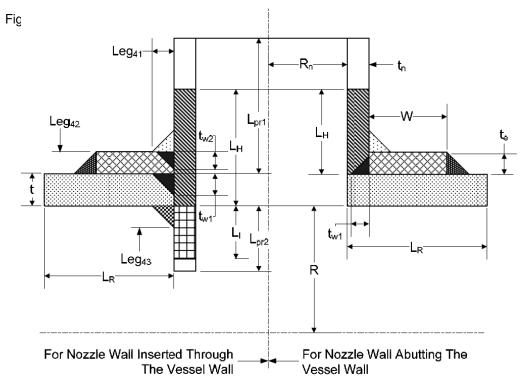
Fig 4.5.9

$$t_{n_int_min} := \frac{MAWP_{pv} \cdot (R_n)}{S_{max_Ti_g3} \cdot E_w - 0.6 \cdot MAWP_{pv}}$$

$$t_{n_int_min} = 0.484 \text{ mm}$$

However, the nozzle not only must withstand internal pressure, but must also help to carry head stresses across the openings made for them.

4.5.10.1 Procedure for Radial Nozzle in a Spherical or Formed head



a) Step 1

$$R_{eff} := L_{cr}$$
 $R_{eff} = 1.14 \,\text{m}$ (4.5.64)

b) Step 2- limit of reinforcement along vessel wall

sec VIII- div. 2 sec 4.3.9.2(d) states that drilled holes for nozzle attachment must be located outside reinforcement area. for NW100 flange (Rayotek #7). This will likely limit the reinforcement area, if not limited by other equations. sec VIII- div 1 rules (UG-40(e) state that flanges attached by studs do not count for reinforcement, even for that part located within the area of reinforcement. sec VII-div 2 rules do not explicitly state this (insofar as I have found), but is is clear, by implication, that this holds as well. Plus, I would not rely on the sapphire window to carry any pressure stresses; it is not a material allowed under the ASME PV rules sec II part D.

NW100 flange dimensions, from Rayotek drawing "Conflat flange External pressure #7 NW100 (we need to verify that these are good for 15 bar ext. pressure, not just vacuum, as they have a larger aperture than ones designed for internal pressure):

bolt circle dia.	bolt hole dia	outer diameter	clear aperture
BC := 13.02cm	BH := 8 4mm	OD := 15.24cm	CA := 8.9cm

Limit of reinforcement along vessel wall:

$$L_{R1a} := 0.5 \cdot BC - (R_n + 0.5BH)$$
 $L_{R1a} = 2.14 \text{ cm}$

Though this is likely the minimum, there are other limits to consider. We must first determine the various D_i that will be present (see fig 4.5.9 above). We can use "vectors" in mathCAD to perform parallel calculations. Three "rings" of flanges will fit if spaced at 8 degree spacing along radius of crown:

$$\mathbf{d_S} \coloneqq \mathbf{L_{CT}} \cdot 8 \text{deg} \qquad \quad \mathbf{d_S} = 15.917 \, \text{cm} \quad \quad \text{Check --> } \quad \mathbf{OD} = 15.24 \, \text{cm}$$

Then:
$$D_{R} := \begin{pmatrix} 0 \\ \sin(8\text{deg}) \\ \sin(16\text{deg}) \\ \sin(24\text{deg}) \end{pmatrix} \cdot L_{cr} \qquad D_{R} = \begin{pmatrix} 0 \\ 15.866 \\ 31.423 \\ 46.368 \end{pmatrix} \text{cm}$$

Possible limits of reinforcment: $L_{R1} \coloneqq 0.5 \cdot D_i - \left(D_R + R_n + t_n\right) \quad L_{R1} = \begin{pmatrix} 53.049 \\ 37.183 \\ 21.626 \\ 6.621 \end{pmatrix} cm$ (4.5.67)<---as we get toward knuckle, limits decrease

$$\sqrt{R_{\text{eff}} \cdot t_{\text{ts}}} = 13.077 \,\text{cm}$$
 $2R_n = 7.9 \,\text{cm}$

$$L_{R2} := \min(\sqrt{R_{eff} \cdot t_{ts}}, 2R_n)$$
 $L_{R2} = 7.9 \,\text{cm}$ (4.5.67)

Final Limit of reinforcement along vessel wall:

$$L_{R} := \min(L_{R1}, L_{R2}, L_{R1a})$$
 $L_{R} = 2.14 \text{ cm}$ (4.5.68)

c) Step 3- limit of reinforcement along nozzle wall projecting outside vessel surface wall.

We have no pad reinforcement, and no nozzle so:

$$t_{e} := 0 \text{mm} \quad L_{pr1} := 0 \text{cm} \quad L_{pr2} := 0 \text{cm}$$

$$L_{H} := \min \left[\left(t_{ts} + t_{e} + F_{p} \cdot \sqrt{R_{n} \cdot t_{n}} \right), L_{pr1} + t_{ts} \right]^{\blacksquare}$$
(4.5.73)

where:

re:

$$X_{0} := D_{R} + R_{n} + t_{n}$$

$$X_{0} = \begin{pmatrix} 0.04 \\ 0.198 \\ 0.354 \\ 0.503 \end{pmatrix} m$$
(4.5.79)

ere:

$$X_{o} := D_{R} + R_{n} + t_{n}$$

$$X_{o} = \begin{pmatrix} 0.04 \\ 0.198 \\ 0.354 \\ 0.503 \end{pmatrix} m$$

$$C_{p} := e$$

$$C_{p} = \begin{pmatrix} 20 \\ 5.331 \\ 1.458 \\ 0.42 \end{pmatrix}$$

$$(4.5.79)$$

$$C_n := \min \left[\left(\frac{t_{ts} + t_e}{t_n} \right)^{0.35}, 1.0 \right]$$
 $C_n = 1$ (4.5.81)

$$F_p := \min(C_n, C_p)$$
 $F_p = 0.42$ (4.5.80)

$$L_H := \min \left[\left(t_{ts} + t_e + F_p \cdot \sqrt{R_n \cdot t_n} \right), L_{pr1} + t_{ts} \right] \\ L_H = 1.5 \, \text{cm} \quad \text{<--we have only the wall} \quad (4.5.73) \\ \text{thickness for a limit}$$

d) Step 4- limit of reinforcement along nozzle wall projecting inside vessel surface wall, if applicable

$$L_{I} := \min(F_{p} \cdot \sqrt{R_{n} \cdot t_{n}}, L_{pr2})$$
 $L_{I} = 0 \text{ cm}$ (4.5.82)

e) Step 5- determine total available area near nozzle opening

(material strength ratios)-->
$$f_{rn} := 1$$
 $f_{rp} := 1$ (4.5.30) (4.5.31)

(material strength ratios)-->
$$f_{rn} := 1$$
 $f_{rp} := 1$ (4.5.30) (4.5.33) $A_T := A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp} \cdot A_5$

$$A_1 := t_{ts} \cdot L_R$$
 $A_1 = 3.21 \text{ cm}^2$ (4.5.84)

$$A_2 := t_n \cdot L_H$$
 $A_2 = 1.5 \times 10^{-3} \text{ cm}^2$ (4.5.86)

$$A_3 := t_n \cdot L_I$$
 $A_3 = 0 \text{ cm}^2$ (4.5.83)

$$A_{41} := 0.5L_{41}^2$$
 $A_{41} = 0 \text{ cm}^2$ (4.5.88)

$$A_{42} := 0.5L_{42}^{2}$$
 $A_{42} = 0 \text{ cm}^{2}$ (4.5.89)

$$A_{3} := t_{n} \cdot L_{1}$$
 $A_{3} = 0 \text{ cm}$
 $A_{41} := 0.5L_{41}^{2}$
 $A_{41} = 0 \text{ cm}^{2}$
 $A_{41} := 0.5L_{42}^{2}$
 $A_{42} := 0 \text{ cm}^{2}$
 $A_{42} := 0 \text{ cm}^{2}$
 $A_{43} := 0.5L_{43}^{2}$
 $A_{43} := 0 \text{ cm}^{2}$
 $A_{44} := 0 \text{ cm}^{2}$
 $A_{45} := 0 \text{ cm}^{2}$

$$A_5 := 0 \text{cm}^2$$
 (4.5.94)

$$A_T := A_1 + f_{rn} \cdot (A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp} \cdot A_5$$
 $A_T = 3.212 \,\text{cm}^2$ (4.5.83)

f) Step 6 determine applicable forces

$$t_{eff} := t_{ts} \cdot \left(\frac{t_{ts} \cdot L_R + A_5 \cdot f_{rp}}{t_{ts} \cdot L_R} \right) \qquad t_{eff} = 15 \,\text{mm}$$

$$(4.5.100)$$

$$R_{xn} := \frac{t_n}{\ln\left(\frac{R_n + t_n}{R_n}\right)} \quad R_{xn} = 3.95 \text{ cm} \quad R_{xs} := \frac{t_{eff}}{\ln\left(\frac{R_{eff} + t_{eff}}{R_{eff}}\right)} \quad R_{xs} = 1.147 \text{ m}$$
 (4.5.98)

$$f_{N} := P \cdot R_{xn} \cdot (L_{H} - t_{ts}) \qquad f_{N} = 0 N$$

$$(4.5.95)$$

$$f_S := \frac{P \cdot R_{XS} \cdot (L_R + t_n)}{2}$$
 $f_S = 1.917 \times 10^4 \,\text{N}$ (4.5.96)

$$R_{nc} := R_n$$
 (radius along chord= R_n for radial nozzles)

$$f_T := \frac{P \cdot R_{xs} \cdot R_{nc}}{2}$$
 $f_T = 3.537 \times 10^4 \,\text{N}$ (4.5.97)

g) Step 7 determine effective thickness for nozzles in spherical, ellipsoidal, or torispherical heads

$$t_{eff} = 1.5 \,\text{cm}$$
 same formula as above in step 6 (4.5.100)

h) Step 8 Determine avg. local primary membrane stress and general primary membrane stress at nozzle intersection

$$\sigma_{\text{avg}} := \frac{f_N + f_S + f_T}{A_T}$$
 $\sigma_{\text{avg}} = 169.8 \,\text{MPa}$
(4.5.101)

$$\sigma_{circ} := \frac{P \cdot R_{XS}}{2t_{eff}} \qquad \qquad \sigma_{circ} = 59.7 \text{ MPa}$$
 (4.5.102)

I) Step 9 Determine maximum local primary membrane stress

$$P_L := \max \left[\left(2\sigma_{avg} - \sigma_{circ} \right), \sigma_{circ} \right]$$
 $P_L = 280 \,\mathrm{MPa}$ (4.5.103)

$$S_{max_Ti_g3_div2} = 190.3 \, MPa \qquad \qquad E_w = 1$$

$$S_{allow} := 1.5S_{max_Ti_g3_div2} \cdot E_w$$
 $S_{allow} = 285.4 \text{ MPa}$ (4.5.43)

j) Step 10 Maximum local primary membrane stress must be less than the allowable stress

$$P_{L} \le S_{allow} = 1$$
 (4.5.104)

k) Determine max allowable working pressure of the nozzle

$$A_{p} := R_{xn} \cdot (L_{H} - t_{ts}) + \frac{R_{xs} \cdot (L_{R} + t_{n} + R_{nc})}{2} \quad A_{p} = 349.5 \,\text{cm}^{2}$$
(4.5.108)

$$P_{\text{max1}} := \frac{S_{\text{allow}}}{\left(\frac{2A_{\text{p}}}{A_{\text{T}}} - \frac{R_{\text{xs}}}{2t_{\text{eff}}}\right)} \qquad P_{\text{max1}} = 15.7 \,\text{bar}$$
 (4.5.105)

$$S := S_{max_Ti_g3_div2} \quad S = 190.3 \,\text{MPa}$$

$$P_{max2} := 2 \cdot S \cdot \left(\frac{t_{ts}}{R_{xs}}\right) \qquad \qquad P_{max2} = 49.1 \,\text{bar}$$
 (4.5.106)

$$P_{\text{max}} := \min(P_{\text{max}1}, P_{\text{max}2})$$
 $P_{\text{max}} = 15.7 \text{ bar}$ (4.5.107)

Conclusion: Sapphire or quartz windows in NW100 CF flanges may be used in a Ti ASME grade 3 torispheric head, 1.14cm ID, (with openings large enough to allow the PMT to fit through the head opening to contact the window), provided head thickness is increased from 7mm (no holes) to 15mm. Flange stud holes (8.4mm dia.) may be drilled no deeper than 11mm, giving 1.2 diameters of thread engagement, if bottom tapped. This should be sufficient for sealing purposes. Similar flanges can be used on outside of head, provided stud holes are staggered from inside holes. Head thickness of 15mm is required at the minimum thickness, after machining for flanges; it is not a stock material thickness, which may be 16mm or more.